2–D DISCRETE WAVELET–BASED CRACK DETECTION USING STATIC AND DYNAMIC RESPONSES IN PLATE STRUCTURES

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ABSTRACT

This paper peresents the 2–D discrete wavelet–based crack detection using static and dynamic responses in plate structures. Over the last few decades, the wavelet–based techniques of damage identification methods have been presented and developed in civil and mechanical structures. The techniques are based on a comparison between the current structural responses as damaged structure and those of the previous baseline state, which is considered to be the health structure. To impliment the damage detection of plate structures, in this study the displacement, rotation and stress as the static responses and the mode shapes as the dynamic responses are considered in the crack detection procedure. The numerical results show that the wavelet–based crack detection is sensitive to number of cracks in plate structure. In other words, the existence of a crack can be potentially identified using each of the static or dynamic responses of plate structures. Moreover, if multi–cracks exists in plate structure, both static and dynamic responses of plate structure should be utilized in the damage detection procedure.

1. INTRODUCTION

The detection of crack–like defects in mechanical systems and civil engineering structures became an interesting research topic and received considerable attention from engineering researchers in the last decades [1, 2]. The necessity of crack detection is important because the performance of structures may change due to a gradual or sudden change in states, load conditions, or response mechanisms. Hence, the non–destructive damage identification methods have been proposed and developed by a number of researchers. The main advantage of the non–destructive damage detection method is to maintain the safety and

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integrity of structures. Most non–destructive damage identification methods are categorized in two groups: 1) Local damage detection techniques; 2) Global damage detection techniques. The methods are based on the changes in the static and dynamic properties of the structure (stiffness, Young's modulus, etc) caused by damages. Some of the damage detection methods is based on the observation of variations in dynamic or static responses of structures. By creating a crack in a structure, the stiffness of structure changes locally and its static and dynamic behavior occurs as a result. Many researchers have investigated the crack detection of structures by using the static and dynamic responses of structures.

The damage detection in plate structures was presented by many researchers in literature. Cornwell et al. [3] introduced a method for detection and location damage in plate–like structures. This method based on the changes in the strain energy of structure. This method only requires the mode shapes of structure before and after damage. Bayissa and Haritos [4] proposed a new damage detection technique in plate–like structures, which is based on the statistical moments of the energy density function of vibration responses in the time–scale domain. Hadjileontiadis and Douka [5, 6] proposed the effective methods for detection of cracks in plate structures based on fractal dimension analysis and kurtosis analysis. Recently, Xiang et al. [7] presented the damage detection in plate structure based on the fourth strain statistical moment (FSSM). The method consists of two steps: damage localization and damage quantification.

The wavelet–based damage detection is introduced as a promising mathematical tool [8-12]. Wavelet transform is mainly attractive because of its ability to compress and encode information, to reduce noise, or to detect any local singular behavior of a signal. The successful application of wavelet transform for the identification of damage in two–dimensional structures has been investigated by many researchers. Wang and Deng [8] introduced a structural damage detection technique based on wavelet analysis of spatially distributed structural response measurements. In this method, using the static displacement field as input for wavelet transform and Haar wavelet coefficients detect the location of damage. Yan and Yam [9] identified damage, using wavelet analysis in composite plates to decompose the dynamic responses. Chang and Chen [10] proposed a technique for structure damage detection based on spatial wavelet analysis. This method only needs the spatially distributed signals (e.g. the displacements or mode shapes) of the rectangular plate after damage. Douka et al. [13] applied 1–D wavelet transform for the detection of different locations in rectangular plate. The vibration modes of the plate are analyzed using the continuous wavelet transform and both the location and depth of the crack are estimated. Rucka and Wilde [14] presented a method to estimate damage location in plate using continuous wavelet transform. Based on this method, the location of the damage is indentified by a peak in the spatial variation of the transformed response. Bayissa et al. [12] proposed a technique based on the continuous wavelet transform to detect damage in a concrete plate model and in steel plate girder of a bridge structure. Huang et al. [15] developed a distributed 2–D continuous wavelet transform algorithm, which can use data from discrete sets of nodes and provide spatially continuous variation in the structural response parameters to monitor structural degradation. Fan and Qiao [2] proposed a two -dimensional (2-D) continuous wavelet transform (CWT)–based damage detection algorithm using ''Dergauss2d'' wavelet for plate structures. Xiang and Liang [16] developed an

approach for detecting multiple damages in thin plates. This method consists of two steps: 1) Detection of damage location. 2) Identification of damage severities. Xu et al. [17] proposed two–dimensional curvature mode shape method based on wavelets and Teager energy for damage detection in plates. The efficiency of the proposed method was demonstrated using finite element simulations and experimentally validated through noncontact measurement by a scanning laser vibrometer. Recently, He and Zhu [18] presented an adaptive–scale damage detection strategy based on a wavelet finite element model for thin plate structures. In this study, equations of motion and corresponding lifting schemes in thin plate structures have derived with the tensor products of cubic Hermite multi–wavelets as the elemental interpolation functions.

This paper focuses on the 2–D discrete wavelet–based crack detection using static and dynamic responses in plate structures. In this study the displacement, rotation and stress as the static responses and the mode shapes as the dynamic responses are selected in the crack detection procedure in order to detect the crack in plate structures. The procedure of crack detection is performed based on using the distribution of coefficients of wavelet transform. In order to assess the apllication and effect of the static and dynamic responses in the crack detection of plate structures, the rectangular plate with multi–dagme in different location of structure is considered. The numerical results indicate that the crack detection based on the wavelet transform is sensitive to number of cracks in plate structure. The existence of a crack in plate structure can be efficiently identified using each of the static or dynamic responses of plate structures. Moreover, if the plate structure with multi–cracks is considered, both the static and dynamic responses of the plate structure should be utilized in the damage detection procedure.

2. WAVELET TRANSFORM

2.1 One-dimensional wavelet transform

Fast Fourier transform (FFT) as an efficient tool has been introduced for finding the frequency components in a signal [19]. The major disadvantage of FFT is that the transform only shows frequency resolution and no time resolution. Hence, this method is not considered as a suitable tool for processing non–stationary signals. The wavelet transform as a powerful tool has been proposed and utilized for these types of signals. The main advantage of the wavelet transform is to perform the local analysis of a signal

Wavelet analysis can be utilized by selecting a basic wavelet function that can be defied as a function of space x or time t. This basic wavelet function which is called the "mother wavelet" $\psi(t)$, is then dilated (stretched or compressed) by a and translated in space by b to generate a set of basis functions $\psi_{a,b}(t)$ as follows [20]:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi(\frac{t-b}{a})$$

(1)

where a and b are the scale and translation parameters, respectively.

The continuous wavelet transform (CWT) of a signal is the sum over all time of the signal $f(t)$ multiplied by a scaled and shifted version of a mother wavelet [20]:

$$C\left(a,b\right)=\frac{1}{\sqrt{|a|}}\int_{-\infty}^{+\infty}f\left(t\right)\psi^{*}\left(\frac{t-b}{a}\right)dt=\left(f\left(t\right),\psi_{a,b}\left(t\right)\right) \tag{2}$$

where ψ^{*} represents operation of complex conjugate of ψ, and $C\left(a,b\right)$ is called the wavelet coefficients for the wavelet $\psi_{a,b}(t)$.

2.2 2-D discrete wavelet transform
In the CWT concept, the transform and scale parameters are continuously changed. By discretizing the parameters a and b, a discrete wavelet transform (DWT) is obtained. Hence, the procedure becomes much more efficient if the dyadic values of a and b are expressed as follows [21]:

$$a=2^{j}\;;\;\;b=2^{j}k\;;\;\;j,k\in Z \tag{3}$$

where Z is a set of integers.
   The corresponding discretized wavelet $\psi_{j,k}(t)$ is as follows:

$$\psi_{j,k}\left(t\right)=2^{j/2}\psi(2^{j}t-k) \tag{4}$$

Also, Equation (2) can be expressed as:

$$C\left(a,b\right)=\left\langle f\left(t\right).\psi_{a,b}^{*}(t)\right\rangle \tag{5}$$

Thus, CWT is a collection of inner products of a signal and the translated and dilated wavelets. In discrete wavelet transform the signals can be defined by approximations and details. The detail at level j is expressed as follows [21]:

$$D_{j}\left(t\right)=\sum_{k\in Z}cD_{j,k}\psi_{j,k}\left(t\right) \tag{6}$$

where $cD_{j,k}$ is wavelet coefficients at level j.
   The approximation at level j is defined as:

$$A_{j}\left(t\right)=\sum_{k=-\infty}^{+\infty}cA_{j}\left(k\right)\phi_{j,k}\left(t\right) \tag{7}$$

where $\phi_{j,k}$ is scaling functions and $cA_{j,k}$ is scaling coefficients at level j.
   The signal $f(t)$ can be represented by:

$$f(t) = A_J + \sum_{j<J} D_j \qquad (8)$$

The 2–D DWT is a development of the 1–D DWT for applying to 2–D data. The 2–D DWT is the sum of three matrices of wavelet coefficients (i.e. vertical, horizontal and diagonal). These three matrices can be defined as the production of wavelet function and scaling function as [21]:

$$\psi^V(x,y) = \varphi(x)\psi(y)$$
$$\psi^H(x,y) = \psi(x)\varphi(y) \qquad (9)$$
$$\psi^D(x,y) = \psi(x)\psi(y)$$

where ψ^V responds to variations along rows, ψ^H measures variations along columns, and ψ^D corresponds to variations along diagonals.

3. RESPONSES OF PLATE STRUCTURES

The damage detection procedure is based on the fact that the loss of stiffness due to damage affects the stactic and dynamic responses of the structure. Therefore, the damage existance of a structure can be detected by monitoring and identifying the static and dynamic responces of a structure. The responces consist of displacement, rotation and stress as the static responces and the dynamic responces such as natural frequencies and mode shapes. In order to obtain the stactic and dynamic responses of a plate structure in the damage detection procedure,the static and dynamic analysis of a rectangular plate with and without crack are determined using finite element method. It is assumed that a four–fixed rectangular plate with a crack shown in Fig. 1 is considered as damaged structure. The dimensions of the plate are $L \times B \times t$. The crack considered in the plate is the linear damage with the length of W and the width of D and located in distance of L_1 and B_1 from the left edge of the plate.
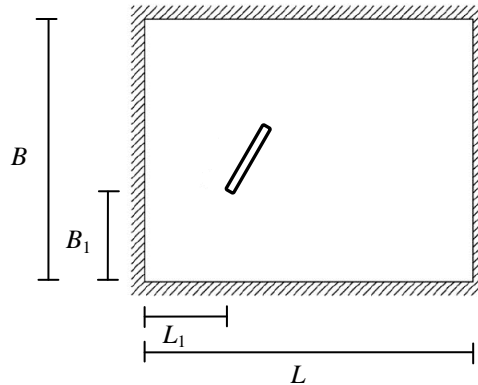

Figure 1. Geometry of the plate structure with linear damage

In this study, the carck in structure is identified by defining the degradation of the Young's modulus in damaged elements as follows:

$$E_d = E(1-d) \tag{10}$$

where E_d and E are considered as the damaged and undamaged Young's modulus of the element, respectively; and d indicates the damage severity at the element of the finite element model.

3.1 Static responses of plate structure

In this study, the static of responses of plate structure is obtained based on the finite element analysis with the Mindlin–Reissner plate theory. In the Mindlin–Reissner theory shear deformations are taken into account, and application of ordinary low–order finite element is not capable to reproduce the pure bending modes in the limit case of thin plate. The shear locking problem arises due to inadequate dependence among transverse deflection and two rotations.

Based on the Mindlin–Reissner theory, the strain energy of plate is given as follows [22, 23]:

$$U = \frac{1}{2}\int_V \sigma_b^T \varepsilon_b dV + \frac{\alpha}{2}\int_V \sigma_s^T \varepsilon_s dV \tag{11}$$

where

$$\sigma_b^T = [\sigma_x \ \sigma_y \ \tau_{xy}] \tag{12}$$

$$\varepsilon_b^T = [\varepsilon_x \ \varepsilon_y \ \gamma_{xy}] \tag{13}$$

are the bending stresses and strains, and

$$\sigma_s^T = [\tau_{xz} \ \tau_{yz}] \tag{14}$$

$$\varepsilon_s^T = [\gamma_{xz} \ \gamma_{yz}] \tag{15}$$

are the transverse shear stresses and strains. The α parameter, also known as the shear correction factor can be taken as 5/6 [24].

The generalized displacements based on finite element method are independently interpolated using the same shape functions (N):

$$w = \sum_{i=1}^{n} N_i \omega_i \quad ; \quad \theta_x = \sum_{i=1}^{n} N_i \theta_{xi} \quad ; \quad \theta_y = \sum_{i=1}^{n} N_i \theta_{yi} \tag{16}$$

where w, θ_x and θ_y are the plate deflection in z direction, the rotations of the normal to the middle plane with respect to axes y and x, respectively.

The strain–displacement matrices for bending and shear contributions are obtained by derivation of the shape functions as follows:

$$[B_b]_i = \begin{bmatrix} 0 & 0 & \dfrac{\partial N_i}{\partial x} \\ 0 & -\dfrac{\partial N_i}{\partial y} & 0 \\ 0 & -\dfrac{\partial N_i}{\partial x} & \dfrac{\partial N_i}{\partial y} \end{bmatrix} \qquad i = 1, 2, ... n \tag{17}$$

$$[B_s]_i = \begin{bmatrix} \dfrac{\partial N_i}{\partial x} & 0 & N_i \\ \dfrac{\partial N_i}{\partial y} & -N_i & 0 \end{bmatrix} \qquad i = 1, 2, ... n \tag{18}$$

Hence, the stiffness matrix of the Mindlin plate is then obtained as:

$$K_e = \frac{t^3}{12} \int_{-1}^{1} \int_{-1}^{1} B_b^T D_b B_f \, |J| \, d\xi d\eta + \alpha t \int_{-1}^{1} \int_{-1}^{1} B_s^T D_s B_s \, |J| \, d\xi d\eta \tag{19}$$

where $|J|$ is the determinant of the Jacobian matrix. D_b and D_s is defined as:

$$D_b = \frac{E}{(1-v^2)} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \dfrac{1-v}{2} \end{bmatrix} \tag{20}$$

$$D_s = \frac{E}{2(1+v)} \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} \tag{21}$$

where v is the Poisson's ratio of plate.

The stiffness matrix of elements of plate structure is computed by numerical integration. In this study, the stiffness integral is solved by considering for the four–node element, 2×2 Gauss points for the bending contribution and 1×1 Gauss point for the shear contribution. By calculating the nodal displacements of plate structure, the linear elastic stress–strain relations in bending are defined for a homogeneous, isotropic material as:

$$\sigma_b = D_b \varepsilon_b \tag{22}$$

while the linear elastic stress–strain relations in transverse shear are defined as

$$\boldsymbol{\sigma}_s = \boldsymbol{D}_s \, \boldsymbol{\varepsilon}_s \tag{23}$$

3.2 Dynamic responses of plate structure

The dynamic responses of plate structure are obtained based on the free vibrations of Mindlin plates. By using the Hamilton principle [25], the equations of motion of Mindlin plate are expressed as:

$$\boldsymbol{M} \, \ddot{\boldsymbol{u}} + \boldsymbol{K} \, \boldsymbol{u} = \boldsymbol{f} \tag{24}$$

where $\boldsymbol{M}$, $\boldsymbol{K}$ and $\boldsymbol{f}$ are the system mass and stiffness matrices, and the force vector, respectively, and $\ddot{\boldsymbol{u}}$ and $\boldsymbol{u}$ are the accelerations and displacements.

Assuming a harmonic motion, the natural frequencies and the modes of vibration are obtained by solving the generalized eigen problem [8]:

$$(\boldsymbol{K} - \omega^2 \boldsymbol{M}) \boldsymbol{\varphi} = \boldsymbol{0} \tag{25}$$

where ω is the natural frequency; and $\boldsymbol{\varphi}$ is the mode shape of vibration.

The mass matrix of the Mindlin plate is obtained as:

$$\boldsymbol{M}_e = \int_A \rho \boldsymbol{N}^T \begin{bmatrix} t & 0 & 0 \\ 0 & \dfrac{t^3}{12} & 0 \\ 0 & 0 & \dfrac{t^3}{12} \end{bmatrix} \boldsymbol{N} \, dA \tag{26}$$

It is noted that in this study the fundamental mode shape of the damaged plate and undamaged plate are utilized.

4. WAVELET METHODOLOGY FOR THE CRACK DETECTION

In the procedure of crack detection, the main purpose for the use of wavelets is based on the fact that the existence of cracks produces small discontinuities in the structural response at the damaged locations. Often these discontinuities cannot be observed by the examination of the structural responses, but they are detectable from the distribution of the wavelet coefficients obtained by the signals of the DWT. The following procedure is proposed to detect crack in a plate structure by the DWT as follows:

- A rectangular plate containing one or multi–damage with arbitrary length, depth and location is considered. ABAQUS software package is employed to build the model of

plate structure in the finite element method and obtain the static or dynamic responses of plate. In finite element model, the damage is represented as the elements with reduced Young's modulus.

- Using the measured structural responses, compute the signals (wavelet coefficients) associated with the DWT. In order words, the first one is the approximate signal; the others are three detailed signals, including the horizontal detailed signal, the vertical detailed signal and the diagonal detailed signal.
- Plot and examine the wavelet coefficients or the detail signals of the DWT. Any signal discontinuity will be detected by the distribution of coefficients on the wavelet coefficients plot.

5. NUMERICAL EXAMPLES

In order to investigate the validity the 2–D discrete wavelet–based crack detection using static and dynamic responses in plate structures, a number of numerical examples are given in this section. Because of the clear and best results of these examples, these examples are selected form a number of examples solved by the authors. In the examples, the fixed support rectangular plate is considered with $250cm$ width, $250cm$ height, and the thickness $5cm$. The material assumed to be steel with Young's modulus $E = 2.1 \times 10^6 \, kg/cm^2$, mass density $0.00785 \, kg/cm^2$, and Poisson's ratio $v = 0.33$. The damage severity, d, at the element of the finite element model is assumed to be equal to 10%. The procedure of the crack detection based on the DWT is implimented by a MATLAB code.

5.1 Example 1: The crack detection using the displacements of plate
The fixed support rectangular plate with the vertical and horizontal linear cracks shown in Fig. 2 is presented as the first example.
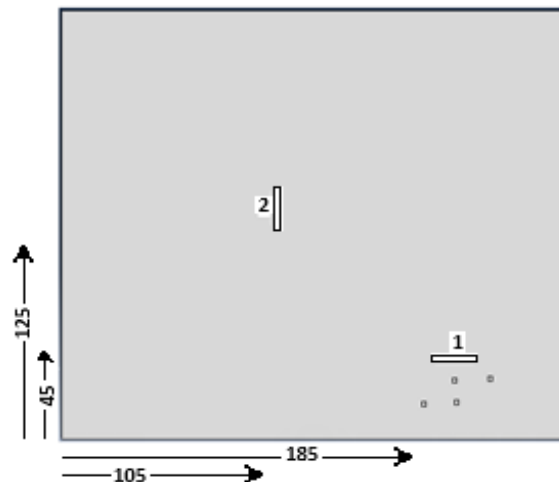

Figure 2. Geometry of the plate structure with the vertical and horizontal linear cracks

The crack with the $20cm$ length and the $3cm$ width is considered in the specify distance of the plate edges, which is depicted on Fig. 2. The four concentrated loads shown in Fig. 2 are applied in the neighbor of the first crack. The example are investigated in two cases for the carck detection using the plate deflection in z direction as the responses of static analysis:
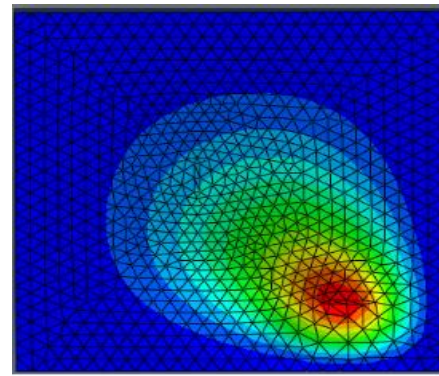
Case 1: Both of cracks are available on the plate structure.

Case 2: Only the second crack is available on the plate structure.

Case 1: The deflection of the plate structure is shown in Figs. 3 and 4 for the undamaged and damged structure, respectivily.
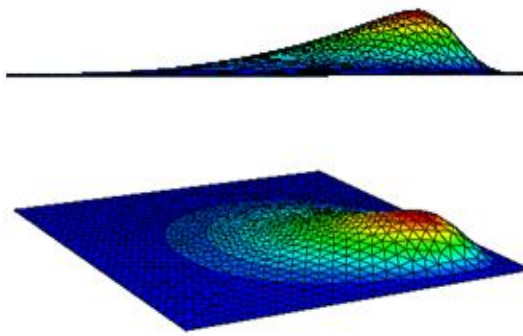
(a) 3-D figure of the displacement   (b) Severity of the displacment

Figure 3. 3-D figure and severity of the displacment in the undamaged plate
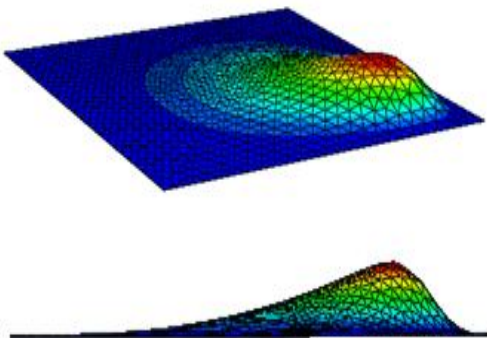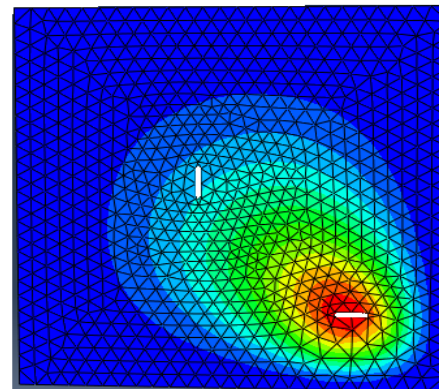
(a) 3-D figure of the displacement   (b) The severity of the displacment

Figure 4. 3-D figure and severity of the displacment in the damaged plate

Figs. 3(a) and 4(a) show the 3-D figure of the deflection in two conditions of the palte. Figs. 3(b) and 4(b) also indicate the severities of the undamaged and damged plate structure, respectivily. As observed from the figures, the most displacement of the plate is created in the location of the first crack. Furthermore, it can be seen from the figures that in the undamaged and damaged plate structure the shape of the structure deflection is same, and a

slight and uniform change happened in the total deflection graph of the structure. Therefore, this small disturbance between the response of the structure in the two conditions is indistinguishable. In other words, comparison between the damaged and undamaged plate shows that the displacement in z direction reveals any local features capable of directly indicating the location or shape (area) of the damages. While by using the wavelet transforms this slight change in the responses of structure is recognizable.

The 2-D DWT is applied to the the displacement of the palte in z direction with a Harr wavelet. The results of the first level of the 2-D DWT has four wavelet coefficients as shown in Figs. 5 to 8. The approximate sub-band of the DWT is plotted in Fig. 5. Furthermore, Figs. 6 to 8 are three detailed sub-bands (or signals) of the DWT, including the horizontal detailed signal the vertical detailed signal and the diagonal detailed signal.
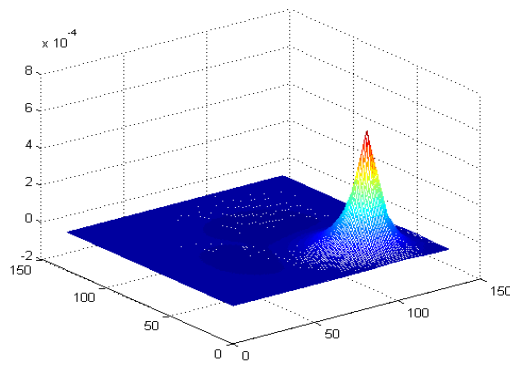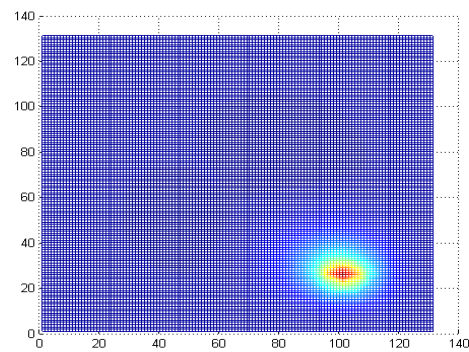
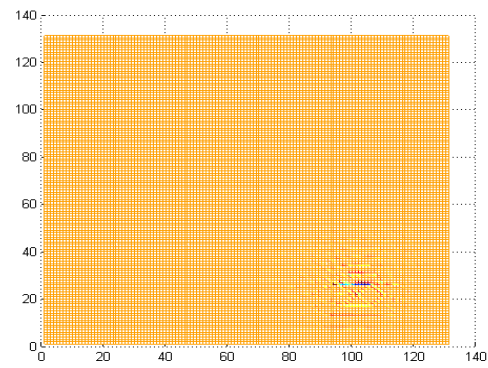(a) Approximate 3-D sub-band   (b) Approximate 2-D sub-band

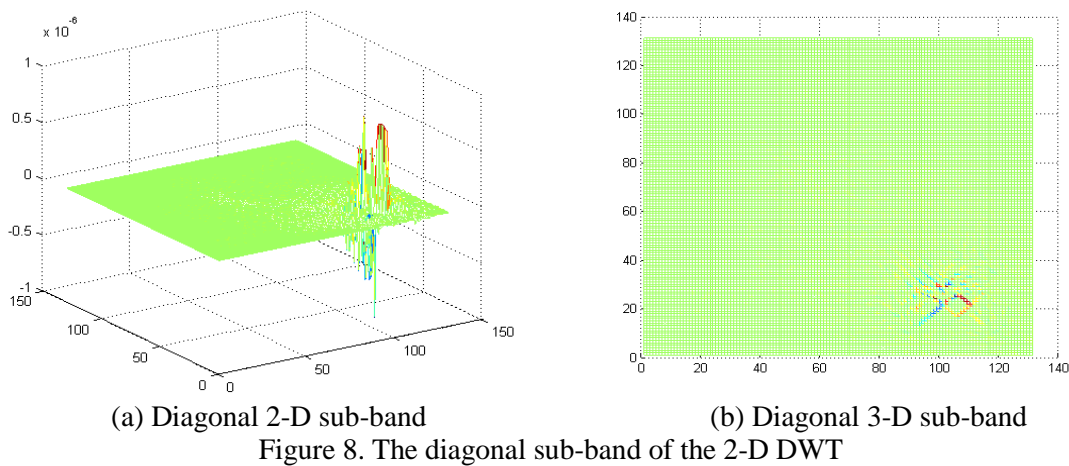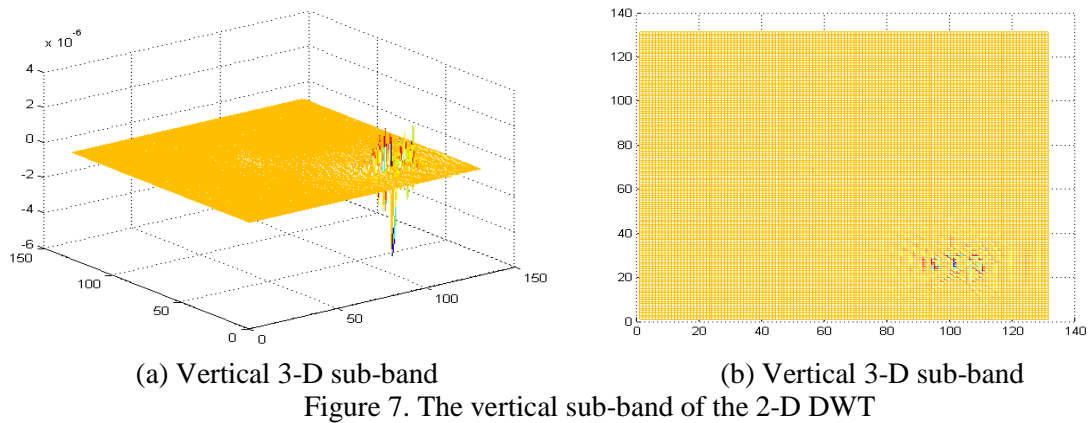Figure 5. The approximate sub-band of the 2-D DWT
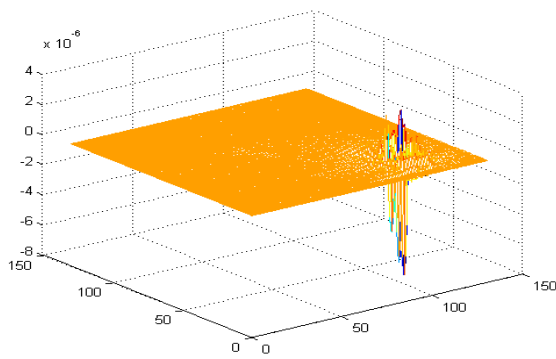
(a) Horizontal 3-D sub-band   (b) Horizontal 2-D sub-band

Figure 6. The horizontal sub-band of the 2-D DWT

(a) Vertical 3-D sub-band   (b) Vertical 3-D sub-band
Figure 7. The vertical sub-band of the 2-D DWT

(a) Diagonal 2-D sub-band   (b) Diagonal 3-D sub-band
Figure 8. The diagonal sub-band of the 2-D DWT

Figs. 5(a) and 5(b) is related to the 2–D and 3–D of the approximate sub-band of the DWT, respectiviely. As seen from Fig. 5(a), only the location of the first crack is clearly identified. In the 3–D of the approximate sub-band of the DWT displayed in Fig. 5(b) a peak is also created in the location of the first crack. Therefore, the first crack located in the region of the plate with the high intensity of displacement can be detected by the DWT while the location of the second crack is the region with the low severity of displacement. In other words, the second crack is impressed by the first crack and cannot detected. For the demonstration of this problrm, the second case is presented in the next section.

Case 2: For this case, the results of a sub-band of the DWT related to the vertical cofficients are shown in Fig. 9. It is noted that other sub-bands of the DWT are as same as the sub-band shown in Fig. 9. It can be seen from Fig. 9(a) that the location of the second crack is clearly identified. By observing the 3–D of the vertical sub-band of the DWT displayed in Fig. 9(b) a peak is also created in the location of the second crack.

(a) Vertical 2-D sub-band   (b) Vertical 3-D sub-band
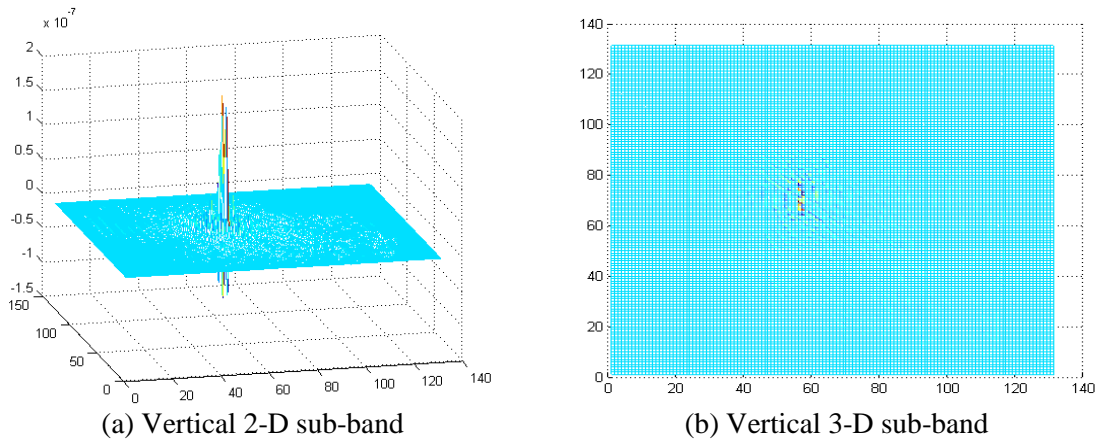
Figure 9. The vertical sub-band of WT

Therefore, in plate structure the detection of cracks depend on the severity of the responses in the location of cracks. It is concluded that number and location of cracks in plate are related to the severity of response in plate structure.

5.2 Example 2: The crack detection using the mode shape of plate

The crack detection procedure of the plate expressed in the first example is implemented by using the fundamental shape mode of plate. Like the first example, the damage detection is investigated in two cases as:

Case 1: Both of cracks are available on the plate structure.

Case 2: Only the first crack is available on the plate structure.

Case 1: The first mode shape of the plate structure is indicated in Figs. 10 and 11 for the undamaged and damged structure, respectivily.

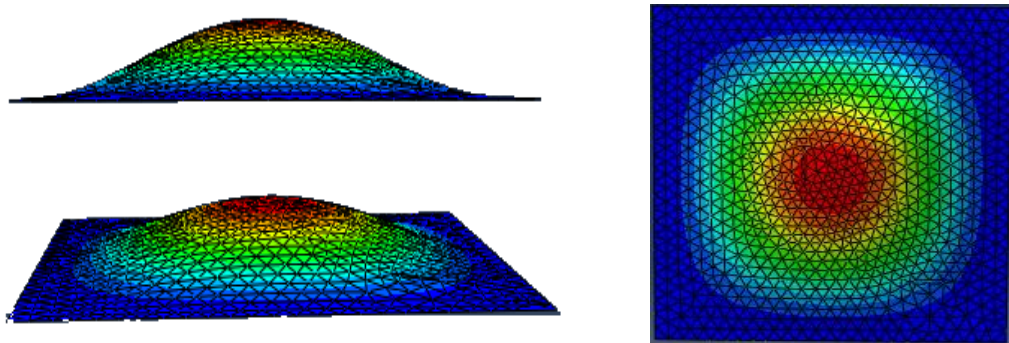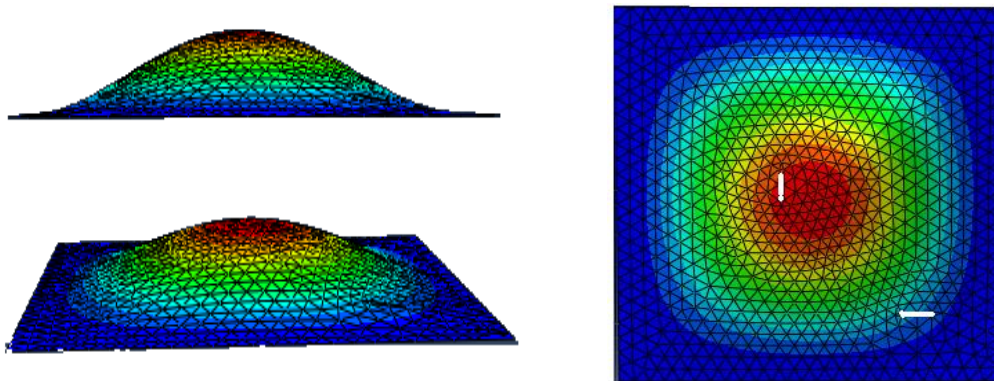(a) 3-D figure of the relative displacement   (b) Severity of the relative displacment

Figure 10. 3-D figure and severity of the relative displacment of the first mode shape in the undamaged plate

(a) 3-D figure of the relative displacement   (b) Severity of the relative displacment

Figure 11. 3-D figure and severity of the relative displacment of the first mode shape in the damaged plate

By comparsion Fig. 10(a) with Fig. 11(a), the most displacement of the mode shape is happened at the middle of the undamaged and damged plate. Since Figs. 10(b) and 11(b) seem completely similar in shape, it is quite difficult too observe the differences. Therefore, it is necessary to have method to find the differences for identification of damage.

In this example, the Haar wavelet is selected to operate as the 2-D discrete wavelet analysis. The results of the DWT of the plate mode shape have two wavelet coefficients as shown in Figs. 12 and 13. Fig. 12 is the approximate sub-band of the DWT, and Figs. 13 is the Diagonal sub-band of the DWT.
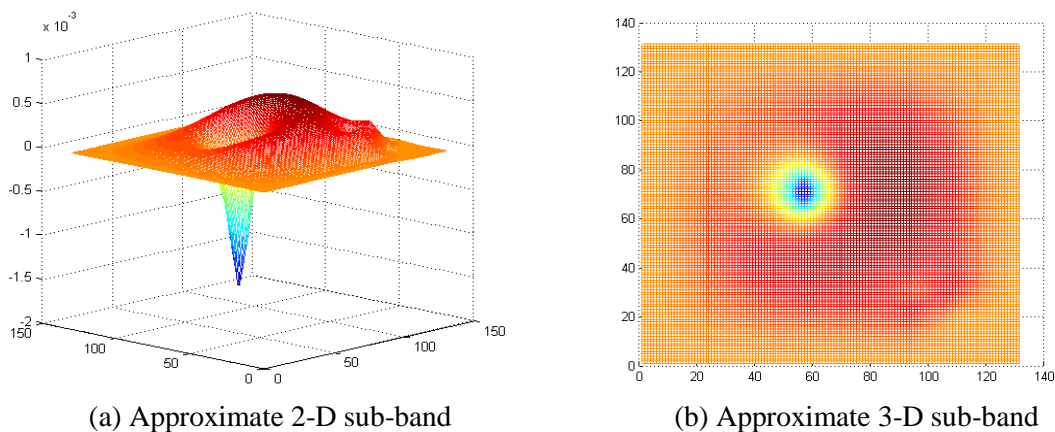
(a) Approximate 2-D sub-band   (b) Approximate 3-D sub-band

Figure 12. The approximate sub-band of the DWT

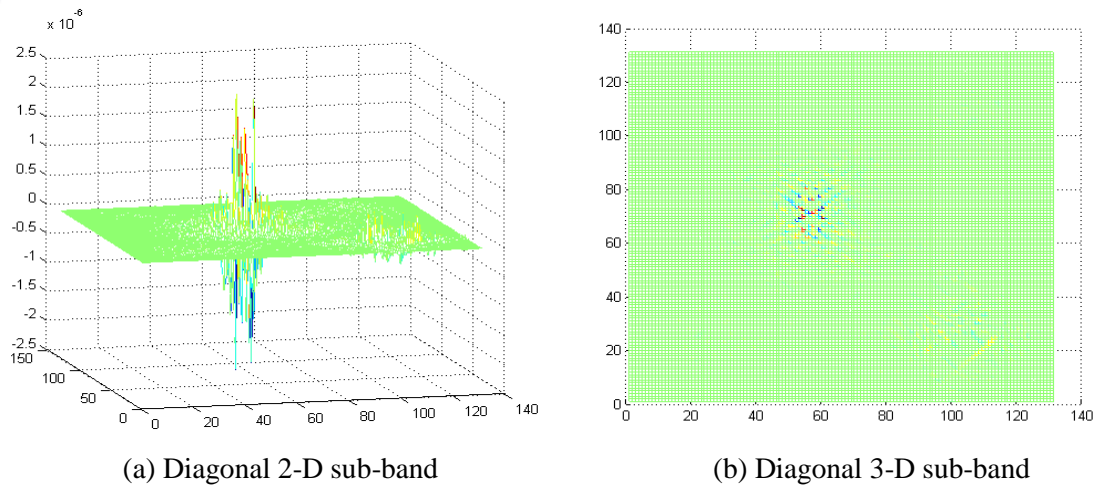(a) Diagonal 2-D sub-band   (b) Diagonal 3-D sub-band

Figure 13. The diagonal sub-band of WT

As shown in Figs. 12 and 13, the second crack can be correctly detected. While the DWT can not identify the location of the first crack. In fact, the second crack is located in the region of the plate with the high severity of the mode shape while the location of the second crack is the region with the low severity of mode shape. Thus, the second crack impresses the fist crack.

Case 2: In second case of damage identification studies, only a single damage is considered in the plate structure. For this case, the results of the first level of the DWT related to the vertical sub-band are shown in Fig. 14.

(a) Horizontal 2-D sub-band   (b) Horizontal 3-D sub-band
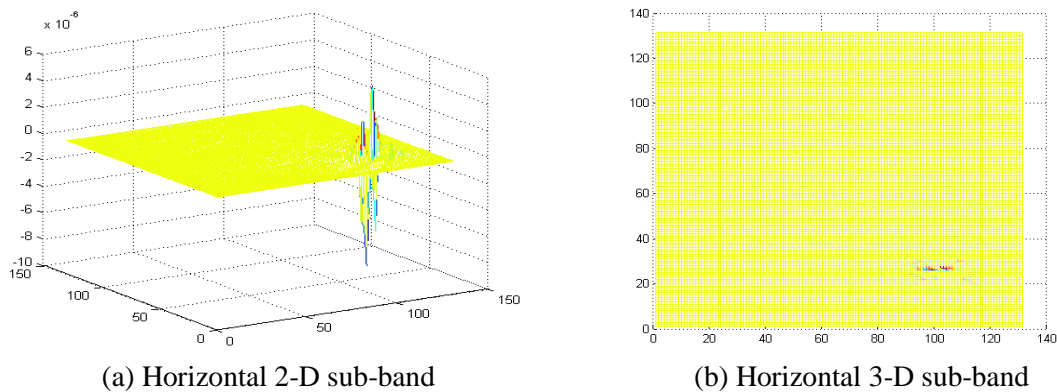
Figure 14. The horizontal sub-band of the DWT

It can be observed from Fig. 14(a) that the location of the crack is clearly identified. As seen in the 3–D of the Horizontal sub-band of the DWT displayed in Fig. 14(b), a peak is also created in the location of the crack. Therefore, for the detection of cracks in plate structure with multi-cracks the responses of two types of plate analysis are often needed.

5.3 Example 3: The crack detection using types of static and dynamic responses
In this example, two linear cracks are induced in plate which is depicted in Fig. 15. The length and width of cracks are equal to $25cm$ and $1cm$, respectively. The location coordinates of the cracks are shown in Fig. 15. The directional angles of the cracks are equal to 0 and $135°$, respectively. The four concentrated loads shown in Figure are applied in near the second crack.
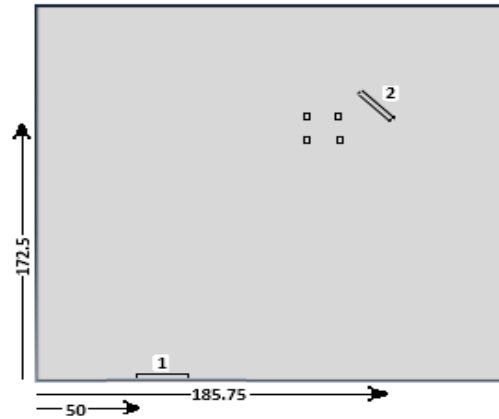

Figure 15. Geometry of the plate structure with mutli-crack

In order to, detect the cracks of the plate structure, the responses of static analysis such as the plate deflection in *z* direction, the displacement, stress, rotations and reactions of plate supports are utilized in the damage detection process. In the following section, the results of damage detection process using the responses is expressed.

5.3.1 Displacement of plate structure
The severity of the displacement in the damaged plate are shown in Fig. 16. It can be observed that the most deflection of plate structure is happened in the location of the loads and near the second crack.
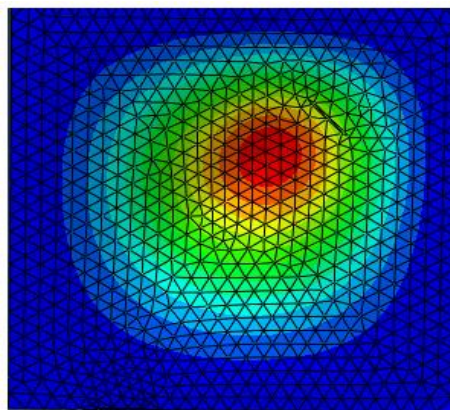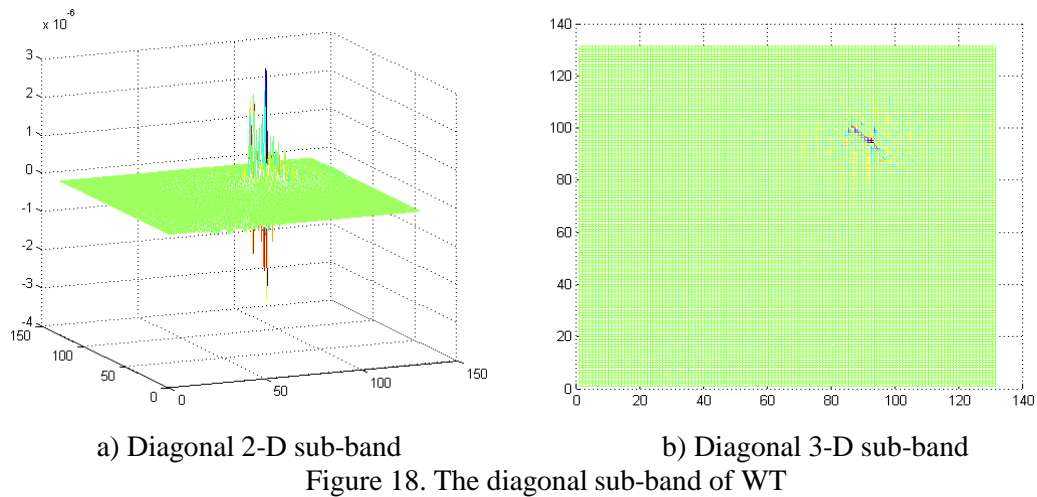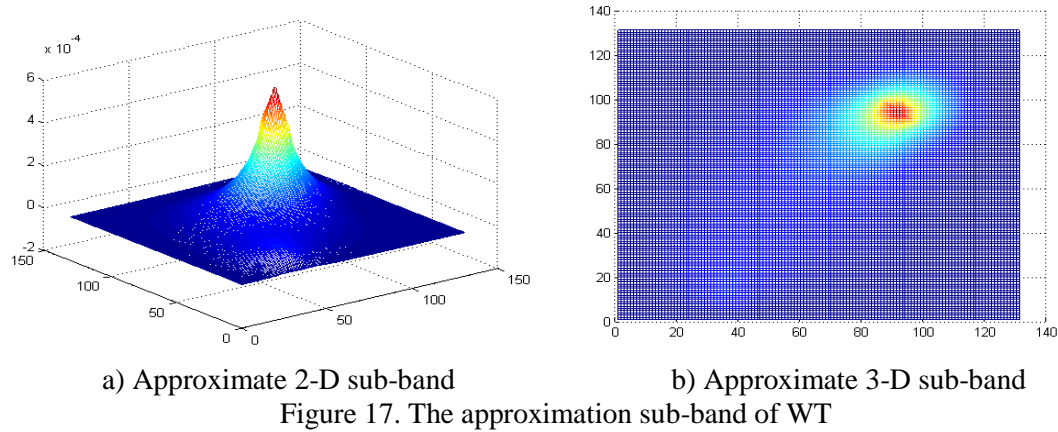

Figure 16. The severity of the displacement in the damaged plate

The results of the 2-D discrete wavelet analysis of the plate deflection are shown in Figs. 17 to 18. Fig. 17 is the approximate sub-band of the DWT, and Fig. 18 is the the diagonal sub-band of the DWT.

a) Approximate 2-D sub-band   b) Approximate 3-D sub-band

Figure 17. The approximation sub-band of WT

a) Diagonal 2-D sub-band   b) Diagonal 3-D sub-band

Figure 18. The diagonal sub-band of WT

It can be observed from Figs. 17 to 18 that the location of the first crack is clearly identified. As seen in the 3–D of the sub-bands of the DWT, a peak is also created in the location of the second crack. Therefore, it is concluded that the first crack is located in the region of the plate with the low severity of displacement while the location of the second crack is the region with the high severity of displacement. The location of the first crack is impressed by the second crack, and cannot be detected by the DWT.

5.3.2 Stress of plate structure

In the second case of damage identification study, the stress in the damaged plate is selected as the static response of the structure. The severity of the stress response in the damaged plate are shown in Fig. 19.
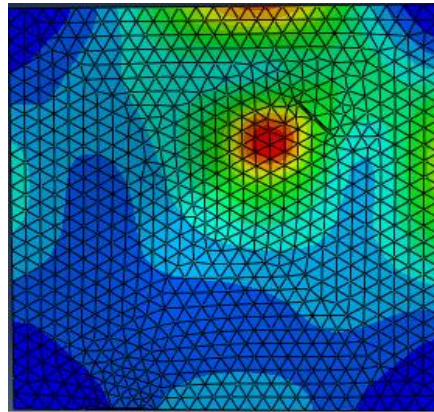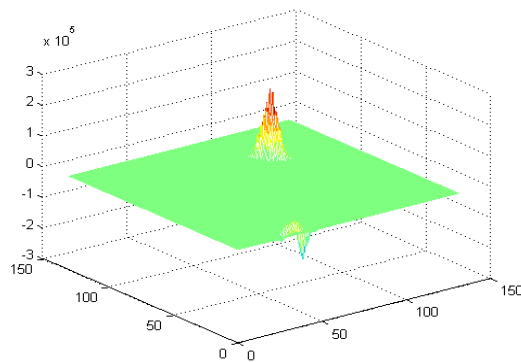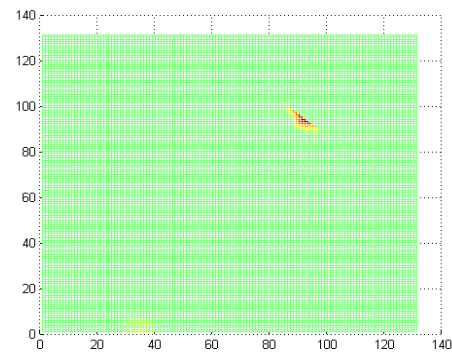
Figure 19. The severity of the stress in the damaged plate

It can be observed from Fig. 19 that the high severity of the sress of the plate structure is happened in the location of the second crack and around of the plate support. Using the response of the plate, the results of the 2-D discrete wavelet analysis are obtained and depicted in Figs. 20 and 21.
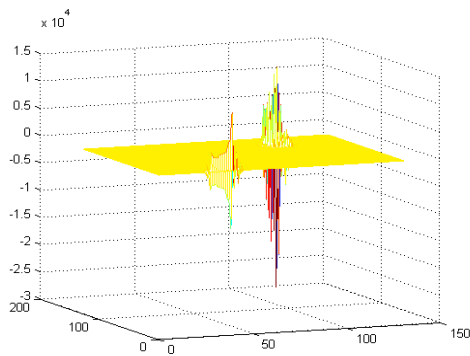

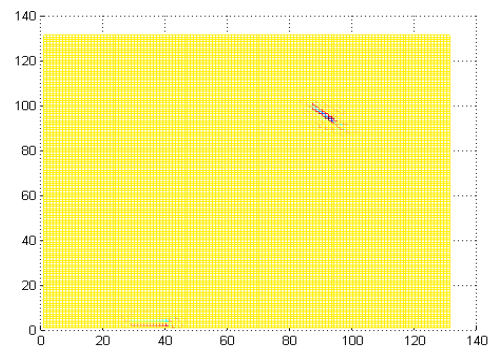(a) Approximate 2-D sub-band   (b) Approximate 3-D sub-band
Figure 20. The approximate sub-band of WT

(a) Horizontal 2-D sub-band   (b) Horizontal 3-D sub-band
Figure 21. The horizontal sub-band of WT

As observed from Figs. 20 and 21, the location of the cracks are clearly detected. It can be seen in the 3–D of the sub-bands of the DWT that a peak is also created in the location of the cracks. Because of the high severity of the stress in the location of the cracks, the cracks can be identified using the DWT. It is noted that the resolution of the DWT sub-bands in the location of the second crack is better than that in the location of the first crack. In fact, in the around of the second the severity of the stress crack in comparsion with that of the first crack is high.

5.3.3 Rotation of plate structure
In the other case of damage identification study, the rotation in the damaged plate is selected as the static response of the structure. The severity of the rotation in the damaged plate are shown in Fig. 22. It can be observed that the most rotation of plate structure is happened the around of the second crack.
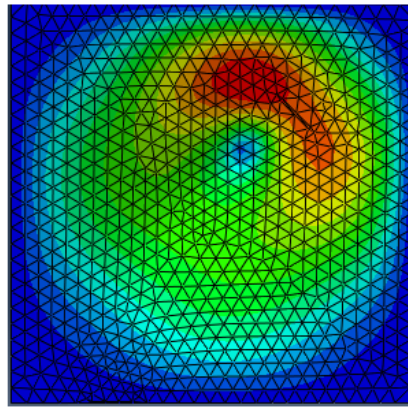

Figure 22. The severity of the stress in the damaged plate

Using the response of the plate, the results of the 2-D discrete wavelet analysis are obtained and depicted in Figs. 20 and 21.
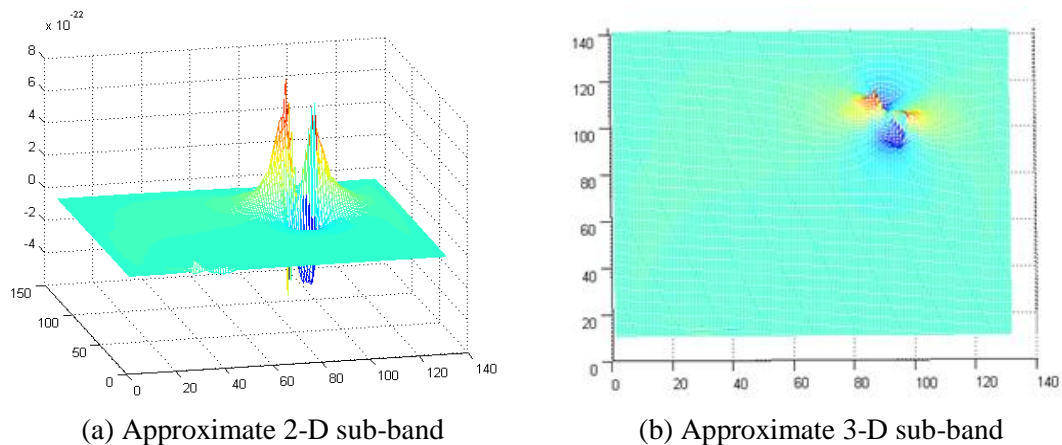

(a) Approximate 2-D sub-band   (b) Approximate 3-D sub-band
Figure 23. The approximate sub-band of WT

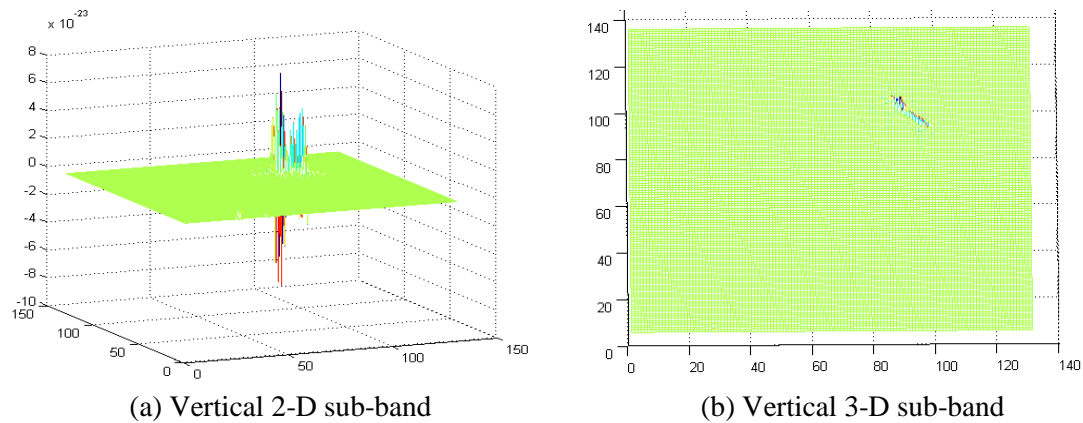(a) Vertical 2-D sub-band   (b) Vertical 3-D sub-band

Figure 24. The vertical sub-band of WT

As observed from Figs. 23 and, the location of the second crack is clearly identified. It can be seen in the 3–D of the sub-bands of the DWT that a peak is also created in the location of the second crack.

5.3.2 Reaction of the support

In the following scenario, the support reaction of the plate structure are selected as the static response. The severity of the support reactions in the damaged palte are shown in Fig. 25.
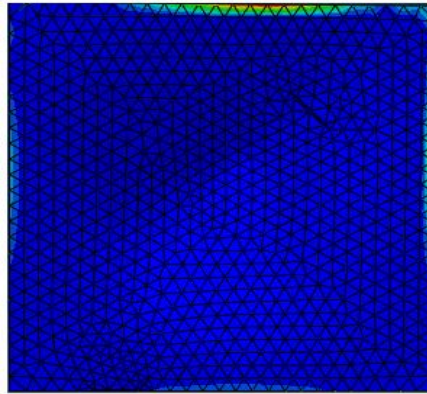

Figure 25. The severity of the support reactions in the damaged plate

As obvious from Fig. 25 the most value of the static response of the plate structure is happened near of the first crack. The results of the 2-D discrete wavelet analysis using the support reaction of the plate shown in Fig. 26.

As can be seen from Fig. 26, the location of the first crack is clearly identified. It can be seen in the 3–D of the sub-bands of the DWT that a peak is also created in the location of the first crack. While the second crack can not be detected by using the support reactions of the plate. In fact, Because the first crack is located near the support of the plate, the DWT can detect this crack.
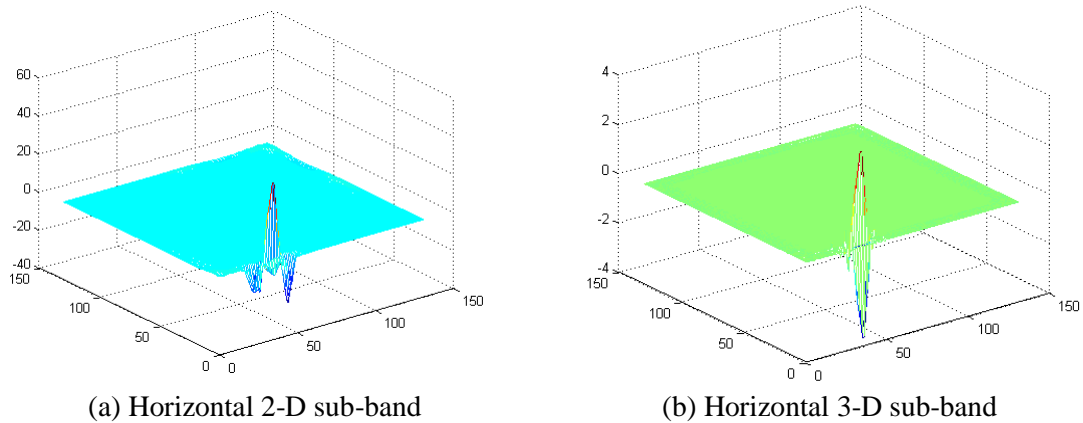
(a) Horizontal 2-D sub-band   (b) Horizontal 3-D sub-band
Figure 26. The horizontal sub-band of WT

6.CONCLUSION

In this study, a 2–D discrete wavelet–based crack detection is presented using static and dynamic responses in plate structures. In order to achieve this purpose, the displacement, rotation and stress as the static responses and the mode shapes as the dynamic responses are selected in the crack detection procedure in order to detect the crack in plate structures. The procedure of crack detection is performed based on using the coefficients distribution of wavelet transform. Based on solving the examples, the following results are obtained:

1. The location of crack in plate structure is important: Based on the location of structural loading in plate structure, the severity of the static responses is different. When crack is located in the region with the most severity of structural responses, the 2-D DWT can clearly indentify the crack and a peack is happened in the location of the crack. This problem can be concluded from the mode shape of plate structure.

2. The number of carcks in plate structure are as an important factor: The existence of a crack in plate structure can be detected based on the 2-D DWT using the static and dynamic responses. If more than a crack are created in plate structure, the detection of carcks depend on the location of cracks. In other words, the crack located in the region with the most severity of responses can be detected and the crack in the region with the less severity of responses is not recognizable.

3. The type of structural response is impotrant in the crack detection process: According to the previous results, it is noted that the severity of the structural responses is different in plate structure. Hence, it is necessary that the different types of plate responces or the reponces of the static and dynamic analysis are utilized in the damage detection procedure.

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